An information-theoretic approach to multi-exposure fusion via statistical filtering using local entropy

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Abstract
An adaptive and parameter-free image fusion method for multiple exposures of a static scene captured by a stationary camera is described. The notion of a statistical convolution operator is discussed and convolution by entropy is introduced. Images are fused by weighting pixels with the amount of information present in their local surroundings. The proposed fusion approach is solely based on non-structural histogram statistics. Its purely information-theoretic view contrasts the physically-based photometric calibration method of high dynamic range (HDR) imaging.

1 Introduction
The dynamic range of light in real world scenes by far exceeds what is capable with a single exposure using a digital camera device. A CCD (charge-coupled device) sensor measures $1 : 10^4$ in contrast but approximately up to $1 : 10^7$ is common in natural scenes [1]. Digital images usually have an 8-bit range of gray values with even lower contrast than original measurements, and therefore a post-processing step built within the camera device applies non-linear dynamic range compression, so that contrast in darker and lighter areas of a scene is lost. The properties of this mapping are physically described by the camera response curve that relates irradiating light to digital gray values. The lack of contrast - due to both the sensor properties and subsequent compression - results in under- and overexposed parts within an image, so that a single exposure is not enough to capture all the details of a scene. Therefore a series of exposures need to be taken and fused via image processing, so that one image contains every detail. The discussion in this paper is restricted to multiple exposures of the same static scene captured with a stationary camera. Otherwise an exposure sequence would need to be registered beforehand, e.g. using the efficient method of [2], and moving objects need to be specially treated, known as ghost removal, which is often done semi-automatically.

1.1 The Physical Approach: Photometric Calibration
The literature discusses two different approaches to fusion of a bracketed exposure sequence of a static scene. The physical approach calibrates an imaging device with respect to its response to different amounts of irradiating light [3, 4, 5, 6]. Thereby the response curve that maps light to digital gray values is recovered. Since the dynamic range of light in real world scenes by far exceeds the usual 8-bit range of gray levels in digital images, the response is likely an S-shaped curve compressing lower and upper bounds of dynamic range. After recovery its inverse is applied and exposures are fused into a single 32-bit floating point radiance map. Because usual display and reproduction devices cannot cope with such images, radiances need to be downscaled again using a tonemapping operator whose compression is adaptive to scene content. Therefore the tonemapped result contains more visual information than is achievable with any single exposure. Although physically sound there are drawbacks. A natural scene used for calibration needs to be carefully chosen, so that it reveals much of the properties of the response curve, and for each image its exposure time must exactly be known. During and after calibration properties of the imaging device, like values for color balancing or sensitivity, are not supposed to change, which is only possible if the device is manually controllable.

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1.2 The Ad-hoc Approach: Exposure Blending

To circumvent these inconveniences another fusion approach has emerged, which abandons the calibration step, therefore allowing to change camera parameters and even incorporate flash images into the to be fused image sequence. Contrary to calibration there is no dynamic range increase possible, because pixels are fused by weighted averaging of gray levels. With methods of this type desirable image qualities are envisioned and measured locally in every exposure image. The fusion result is made up of image patches copied from exposures where the quality measure is maximal. Then exposures with locally non-maximal quality measures are blended in to smooth sharp edges between differently exposed neighbouring patches in the fusion result. The fusion process is a weighted averaging of pixels of a stack of input images guided by quality measures.

1.3 The Proposed Information-Theoretic Approach

Grounded on these ideas, the purpose of this paper is to develop a purely information-theoretic view of multi-exposure fusion that is contradictory to the physically-based calibration method. Here the ad-hoc approach is generalized into entropy-based pixelwise weighting with adaptive scale that is not biased by any spatial gray value pattern that may be preferred by a local quality measure, nor is it based on Gaussian scale-space that inherently spatially contextualizes a local neighbourhood, and neither a distance based weighting function for smooth blending is required. Thereby entropy is the most unbiased statistical measure [7] with respect to the observed data. In image processing measuring entropy equals to a non-structural histogram analysis. Although the maximum entropy method (MEM), that optimizes for the best result achievable using available information only [7], has been previously applied to e.g. multi-spectral image fusion [8] for resolution enhancement the method presented here is not an entropy optimization method but rather a direct convolution approach with entropy as a filter kernel that results in an acceptable but not necessarily optimal outcome in the sense of least biased inference and a priori knowledge.

2 Previous Work

Examples of the second class of fusion algorithms previously introduced are briefly reviewed, where the method presented here is loosely based upon. It is emphasized that any one of those quality measures or blending methods used incorporates some form of structural information and therefore is not purely based on image statistics.

In two of the earlier works images are fused by analyzing the feature space of Laplacian pyramids [9, 10]. For each pyramid level a pattern selective fusion process computes a feature saliency map, e.g. measuring local gradients. Then a pyramid of coefficients is obtained by a selection process on saliency maps that favours images that maximize a composite feature saliency. The fused result is reconstructed from the original pyramids subject to the generated coefficients.

Desirable image qualities defined in [11] are contrast, saturation and well-exposedness. These pixel-wise measures are transformed into single scalar values that compose a weight map corresponding to each exposure. The fusion result is obtained by blending input images with their weight maps. With this naive per-pixel blending disturbing seams appeared in the output where weight maps had sharp transitions due to different absolute intensities caused by different exposures. Smoothing weight maps through Gaussian or bilateral filtering produced other artefacts. To avoid introducing false edges in originally homogeneous areas a multiresolution approach using Laplacian pyramid decomposition is applied. Each input image is decomposed and has weight maps computed at each scale. Then blending is carried out for each level separately, while the Laplacian pyramid is collapsed to obtain the fusion result.

In the work of [12] images are divided into rectangular blocks. For each block and exposure its corresponding entropy is computed. Then the exposure with maximal entropy is selected. After every block is linked to its best input image blending functions propagate with decreasing weights starting with unity from the center of each block over the whole output image to perform weighted averaging of selected input images. The block size and the width of the blending filter are free parameters, which are optimally estimated using gradient descent iteration. A similar region-based method with spatial Gaussian exposure blending is described in [13]. Whereas previously block and filter sizes for blending were globally equal, these are locally adaptive to scene content here using two-dimensional quad-trees to iteratively derive blocks while finer resolution is needed. Besides entropy also intensity deviation and level of detail, that is gradient frequency, are additionally considered as quality measures of a region. Another similar work [14] uses only the level of detail measure with fixed region sizes.

3 Proposed Method

Most of the previous works use a local quality measure of properties that their authors think would well describe a best exposed image. These are generally local gradient frequencies or intensity variances and are supposed to be large in correctly exposed regions because they reveal structure which is not present in the relatively homogeneous over- or underexposed parts of an image. In fact one doesn’t really know what features are worth preserving in an exposure, except that one wants to maximize
information content of an image or make it look interesting. Also blending functions used in previous algorithms do not have a direct connection with image content. These are either explicitly modeled as continuously decreasing Gaussian weight distributions or are inherently present in the Laplacian pyramid approach. But there is no justification other than being smooth and therefore avoiding the creation of artificial edges due to local intensity variations between different exposures within otherwise homogeneous regions.

### 3.1 Exposure Blending based on Local Entropy

To overcome these issues the method presented uses entropy as a measure of information, only. Entropy has already been used in [12], but here it is proposed to use entropy for blending, too, which ultimately means that the fusion result is the pixel-wise average of all input images weighted by their ambient information content. Therefore it becomes now necessary to compute the entropy measure for a local neighbourhood of every pixel per exposure, whereas the previous method analyzed pixel-wise average of all input images weighted by their ambient information content. Therefore it becomes now necessary to use entropy for blending, too, which ultimately means that the fusion result is the scalar gray value intensity images, which are two-dimensional matrices and are denoted by capitalization, e.g. $E$, whereby computations are local using element-wise assignments with the matrix element denoted by $e(x, y)$ correspondingly.

1. Iterate through all stacked exposures $E^{n=1,...,N}$. If $E^{n}$ are in color, convert them into their single-channel luminance representation $L^{n}$, otherwise set $L^{n} = E^{n}$. Because in real world images color channels are expected to be highly correlated [15], it is justified to measure entropy of the luminance image, only.

2. Define the probability $p$ for a specific gray value $g$ to occur in image $n$ of the stack within a particular square region that depends on its location $(x, y)$ and is bounded by its width $b(x, y)$ as

$$p_{g}^{n}(x, y) = \frac{\sum_{i,j=-b(x, y) - 1}^{b(x, y) - 1} \delta_{g}(l^{n}(i, j))}{b(x, y)^2}$$

whereby the delta function $\delta_{g}$ counts the occurrences

$$\delta_{g}(l) = \begin{cases} 1 & l = g \\ 0 & \text{otherwise} \end{cases}$$

Then for each $L^{n}$ compute its corresponding entropy image $S^{n}$ using Shannon’s definition, whereby a pixel

$$s^{n}(x, y) = -\sum_{g=0}^{255} p_{g}^{n}(x, y) \cdot \log_{2}(p_{g}^{n}(x, y))$$

measures the information content of $L^{n}$ within a squared region $b(x, y)$ centered at location $(x, y)$.

3. Fuse all input images $E^{n}$ into the resulting image $R$, which is the sum of all $E^{n}$ weighted and normalized by their corresponding entropy images $S^{n}$,

$$r_{\bullet}^{*}(x, y) = \frac{\sum_{n=0}^{N} s^{n}(x, y) \cdot e_{\bullet}^{n}(x, y)}{\sum_{n=0}^{N} s^{n}(x, y)}.$$  

If $E^{n}$ are multi-channel images, then the same weights from the single-plane weight maps $S^{n}$ are applied to each channel separately, i.e. in the above formula iteratively replace $\bullet$ with R, G, B planes for color images or either set $\bullet = I$ to fuse scalar gray value intensity images $E^{n}$.

### 3.2 Concept of Non-Structural Statistical Convolution

The usage of entropy in this paper is much like in the sense of a convolution operator: At every pixel location entropy is measured over its surroundings and the result is stored for that pixel. Mostly, filter kernels are a pattern of weights and the filter result is a weighted sum of pixel values that is proportional to the features one wants to detect or enhance. Thus the filter result carries structural information about the surroundings due to the pattern of weights.

On the other hand there are filters that do not depend their output on where a specific gray value is found, but on the histogram statistics of the distribution of gray values itself without weighting pixels due to their distance from the center of the filter kernel. Hence, these filters do not have a pattern. Known filter kernels in this sense are the mean and the median operator. These compute their result from local histogram analysis, and may therefore be classified as non-structural statistical convolution operators.
Here entropy is used as a non-structural statistical convolution operator: A measure of information is computed from a local histogram, that describes the level of uncertainty about its distribution. With increasing uncertainty the filter response increases, too. Entropy of a histogram has its maximum when the probability of occurrence of any gray value is equal, and its minimum when there is only a single gray value possible. This is intuitive, because when every gray value is equally possible for 8-bit images one has to ask 255 questions in the worst case to finally know the value of a certain pixel, but if it is known that there is only a single value possible, then the one and only question is which one, and therefore certainty is high. Note that the filter response does not make any proposition about the actual spatial gray value pattern the histogram stems from.

The purpose of the entropy filter here is to detect the amount of activity at a pixel within a certain exposure. In previous works on exposure blending the activity measure has been defined by extend and frequency of gray value gradients that impose a preferred spatial structure of image content. By using entropy no features like gray value edges are preferred, but the only feature is the interest in a certain pixel which becomes greater with increasing uncertainty about its surroundings when spatial correlation is assumed from a-priori knowledge. For example, an image region which is made up of two different gray values and either shows two homogeneous parts separated by a single bar or otherwise a speckle pattern would give different results using gradient-based activity detection, where the speckle pattern would be preferred because there are more gradients, although this might be regarded as noise by most humans. For entropy both spatial patterns are equally interesting, which makes sense, because it cannot distinguish valuable information from noise.

### 3.3 Convolution by Entropy with Fixed Filter Size

Exposure blending based on local entropy has been applied to the exposure series of thirteen images obtained from [16] and shown partly in figure 5. The size of the square filter region is set to $b(x, y) = 17$ pixels for all $(x, y)$ independently. This value was chosen because then the filter has $17 \times 17 = 289$ underlying pixels, so that in an 8-bit image every gray value has a chance to appear, and nevertheless the filter can be centered. The fusion result obtained is shown in figure 1. There are a lot of halo artifacts visible at the borders of objects, which is very common with exposure blending algorithms [11]. These artifacts are thought to exist because of sharp variation in gray value intensities through the exposure series due to bright light sources in the scene. This is confirmed by this work because scenes like figure 6 with smoother intensity distributions do not produce halos using the same filter as above. During test runs with increasing filter widths it was found that halos disappear from brighter regions, but at the same time lower lighted regions became blurry in the fused result. Hence, it has been concluded that the size of the entropy filter needs to locally adapt itself to large intensity variations.

### 3.4 Convolution by Entropy with Adaptive Filter Size

In order to prevent halo artifacts in the fusion result the pixel weighting process needs to integrate entropy over a larger scale window when the brightness variation is large. This finding is reasonable if spatial correlation of brightness is assumed. Then from a large brightness variation at a pixel it can be concluded that its local neighborhood at least is nearly saturated in the longer exposures of the scene. The entropy of a saturated neighborhood is small, because it has a homogeneous appearance and therefore certainty about the measurement is high. In turn certainty of measurements in shorter exposures is low (and entropy is high) due to sensor noise. Therefore noise is more prominently weighted in these areas as is noticeable from the
fused image shown in figure 1 (especially at the window frame and the wooden plate of the desktop). In order to absorb this effect the filter window needs to be larger over those areas, so that statistics from hopefully non-saturated surroundings can be integrated to obtain more meaningful weights. On the other hand if non-saturated measurements are locally available over the whole image set, the filter window should be small, because if it were larger uncertainty would not degrade as fast as possible with longer exposures, since uncertainty is obviously more likely if the integration area is larger. Hence in the fusion result details may be blurred because slightly overexposed pixels would still receive high weights.

In order to define an adaptive integration scale of the entropy filter that depends on absolute brightness variation at the pixel of the filter location the following strategy is proposed.

1. An image $L^{diff}$ that defines the absolute brightness variation of the scene by iterating through the stacked exposures $L^n$ is given by

$$l^{diff}(x, y) = \max_{0 \leq n < N} L^n(x, y) - \min_{0 \leq m < N} L^m(x, y).$$

The result $L^{diff}$ is qualitatively similar to the image of the longest exposure, but e.g. pixels that are continuously saturated are black here, which makes sense, since independent of the filter size a normalized weighting of intensity values always gives the same saturated result. On the other hand pixels that are saturated in the longest exposure become non-saturated here if these are still not underexposed in the shortest exposure. This definition of brightness variation also guarantees that the fusion algorithms does not depend on the order of images in the stack.

2. Because artifacts result from sharp variations in brightness differences present within the scene, the standard deviation of a pixel brightness with respect to the overall range of brightness of the scene is of interest. The variance image $L^{var}$ of the absolute brightness differences $L^{diff}$ is

$$l^{var}(x, y) = \sqrt{(L^{diff} - l^{diff}(x, y))^2}$$

where $L^{diff}$ denotes the mean of $L^{diff}$. An example of a variance image of absolute brightness variation throughout the scene from figure 5 is shown in figure 2. Please note that the resulting image has large gray values at pixel locations where brightness over all exposures is continuously relatively dark, where pixels are underexposed and suffer from thermal noise, or light, where pixels do not contain valid information about the scene because they are saturated. Both cases benefit from larger integration scales, because under the assumption that brightness values are spatially correlated their near neighbourhood does not contain valuable information for fusion by weighted averaging.

3. The filter size $b(x, y)$ should be some function of $l^{var}(x, y)$ to be adaptive to scene content as discussed, and has been chosen to be simply

$$b(x, y) := l^{var}(x, y).$$

Filter results show that this relation is appropriate although other (non-linear) solutions might perform better. E.g. one could additionally cutoff the maximum filter width for faster computation, possibly risking recognizable artifacts in the fusion result.

4 Results and Evaluation

The proposed information-theoretic fusion approach with adaptive filter size is compared to the physically-based calibration method for high dynamic range imaging using the algorithm from [4] with adaptive logarithmic tonemapping [17] implemented by the picturenaut software [18] and the ad-hoc Gaussian scale-space approach described in [11] that is implemented by the enfuse software [19]. The three approaches are qualitatively evaluated on four high dynamic range scenes.

4.1 Qualitative Analysis of Fusion Results of Sample Scenes Comparing Different Methods

Sample images of an exposure series and results of the tonemapped HDR image, the enblend software, and the proposed method are shown for each of the four scenes in figures 5, 6, 7, and 8, respectively. It can be concluded from visual inspection that the proposed approach produces results that are perceptually natural without recognizable artifacts. Also it performs equally well on all example scenes, whereas the tonemapped HDR image in figure 6 has reddish colors and the mecbeth color chart in figure 7 has a foggy appearance. The ad-hoc method implemented by the enfuse software produces visible halo artifacts around the backdrop of the bright light source in figure 7 and produces non-white colors at the window frame in figure 5. With the proposed approach colors of the mecbeth chart in figure 7 are not that vivid as with the enfuse software, but due to the very bright light source this makes a natural appearance to a human viewer. The shortcomings of the tonemapped HDR images are
due to the specific tonemapper used. The problem with unnatural colors in the enfuse algorithm can be explained with the fusion method that prefers mid-range gray value intensities.

For a quantitative analysis entropy has been computed for each fusion result over the whole image once. From the overall discussion in this paper it can be concluded that a higher entropy which corresponds to increased uncertainty in an image is preferable because then it contains greater activity and hence more detail. Quantitative comparison of fusion results is difficult due to the lack of an appropriate metric. One has to keep in mind, that there is also higher entropy in an image if it contains artifacts introduced by the fusion algorithm itself. The results are given in table 1, where entropy measurements from extremal images of the original exposure scene have been included for orientation. If the fusion algorithm is successful it should contain more information than any other of the original images within the series, which is even not true for the memorial and sunset scenes, maybe due to their vast range of radiances in the order of more than five magnitudes. From the quantitative results it is apparent that no single algorithm performs best for all scenes although the tonemapped HDR image seems to be preferable from that point of view, since the sunset scene is the only one where the enfuse algorithm performs better. As already noted during qualitative analysis the mecbeth scene is well fused by the proposed method, but for all other scenes it performs worst although within reach of the ad-hoc approach implemented by the enfuse software.

It is noted as a remark that there has been a class project [20] (the scene used in figure 8 has been made available there) where the HDR method and other ad-hoc approaches are qualitatively compared. The project concludes that ad-hoc methods produce perceptually better results and are more robust.

### 4.2 Physically-based High Dynamic Range Recovery vs. Information-Theoretic Weighting by Entropy

Because the radiance image with increased dynamic range is not directly displayable a false-color image of 32-bit radiances is given in figure 3 corresponding to the scene shown in figure 5. In the spirit of statistical mechanics the aim is to compare the result obtained through physically-based considerations - like calibrating the response curve of a physical system like the camera.
Figure 3. A false-color image showing relative radiance values of the luminance version of the radiance map corresponding to the desktop scene. Radiances have been logarithmically scaled and span over four orders of magnitude.

Figure 4. A false-color image showing accumulated entropies that have been summed pixelwise over all filtered entropy images corresponding to every luminance image of the exposure series of the desktop scene. Entropy values are linearly scaled and represent the amount of information present within a local neighbourhood for every pixel, where the size of the neighbourhood spatially varies but is constant over time represented by the exposure stack.

device - with the information-theoretic result obtained only through analysis of gray value measurements without modelling knowledge of the physical system generating those measurements [7]. Therefore for comparison a similar false-color image is shown in figure 4 that is the accumulation of entropy values obtained by the proposed entropy filter for every pixel throughout the exposure series. Accumulated entropy is expected to approximately correspond to radiance values, because at regions where radiance is high scene details should have been measured at most of the shorter exposures and therefore accumulated uncertainty is high, whereas at image regions with lower radiance only in longer exposures details are visible and with shorter exposures those regions become more homogeneous due to being underexposed, and thus receiving lower entropy values. Also the integration scale for higher radiances is larger, so there is higher probability for uncertainty, and hence entropy is higher. This expectation can be roughly verified by comparing figures 3 and 4. Since entropy is measured over a neighbourhood it is with much less detail than the radiance map, but even some tree leaves can be recognized. The overall energy distribution is valid too, although there is less detail revealed at regions of higher energy due to more aggressive smoothing because of the larger filter scale.

5 Conclusion

In this paper previously developed ad-hoc fusion algorithms for multiple exposure fusion by different authors have been discussed. It has been shown that their fusion approaches are biased by the way how certain image features are preferred when using specific cost functions for exposure selection and blending. Here the ad-hoc approach has been refined into an information-theoretic framework using local entropy for pixelwise averaging that weights pixels by their ambient information content. Because entropy is based on histogram analysis no specific spatial pixel pattern is unjustifiably preferred. The only bias that remains is the integration scale of the entropy filter which has been proven by example to be locally dependend to scene brightness. Therefore a non-structural statistical convolution filter based on local entropy has been newly developed. A method to determine the filter size solely by analysing gray value statistics coupling mean global brightness variation of the scene with
local brightness variances at a single pixel has been introduced, whereby the filter size is different per pixel and depends linearly on brightness variances. It is interesting to note that in terms of statistical mechanics macroscoping and microscopic behavior is linked here. Although a priori knowledge has been applied here, that assumes spatially correlated brightness and an existing relation between integration scale and brightness variance, the filter size is still derived by data-driven gray value histogram analysis. Hence, the whole fusion process is based on information found through histogram analysis, only. The proposed method has been compared to previously developed methods that are representative for physically-based HDR imaging and ad-hoc exposure fusion. Although a qualitative analysis of the proposed method is encouraging, a simple quantitative analysis does not favour any one algorithm under consideration. The presented method is theoretical interesting but a disadvantage is its huge computational cost. Depending on the overall filter sizes processing times are up to twenty minutes on a Pentium IV 2.2 GHz using a single-threaded implementation. However it has applications in information retrieval and visualization, remote sensing and automatic unsupervised blending of exposure bracketed photographs by artists.

References


Figure 5. On the left are samples of 13 exposures. Then fusion results of the HDR, enblend, and the proposed approach follow.

Figure 6. On the left are samples of 16 exposures. Then fusion results of the HDR, enblend, and the proposed approach follow.

Figure 7. On the left are samples of 12 exposures. Then fusion results of the HDR, enblend, and the proposed approach follow.

Figure 8. On the left are samples of 5 exposures. Then fusion results of the HDR, enblend, and the proposed approach follow.